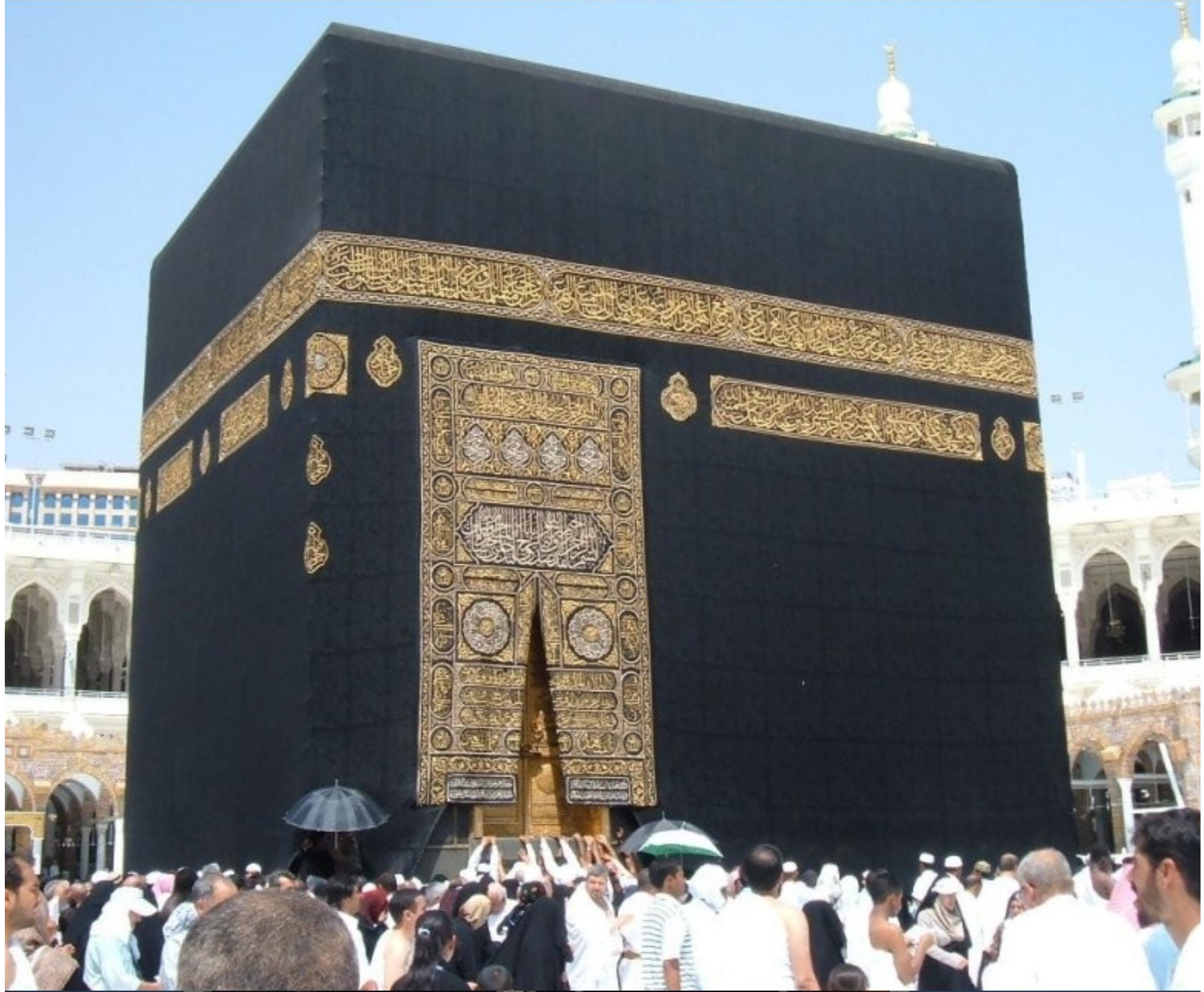
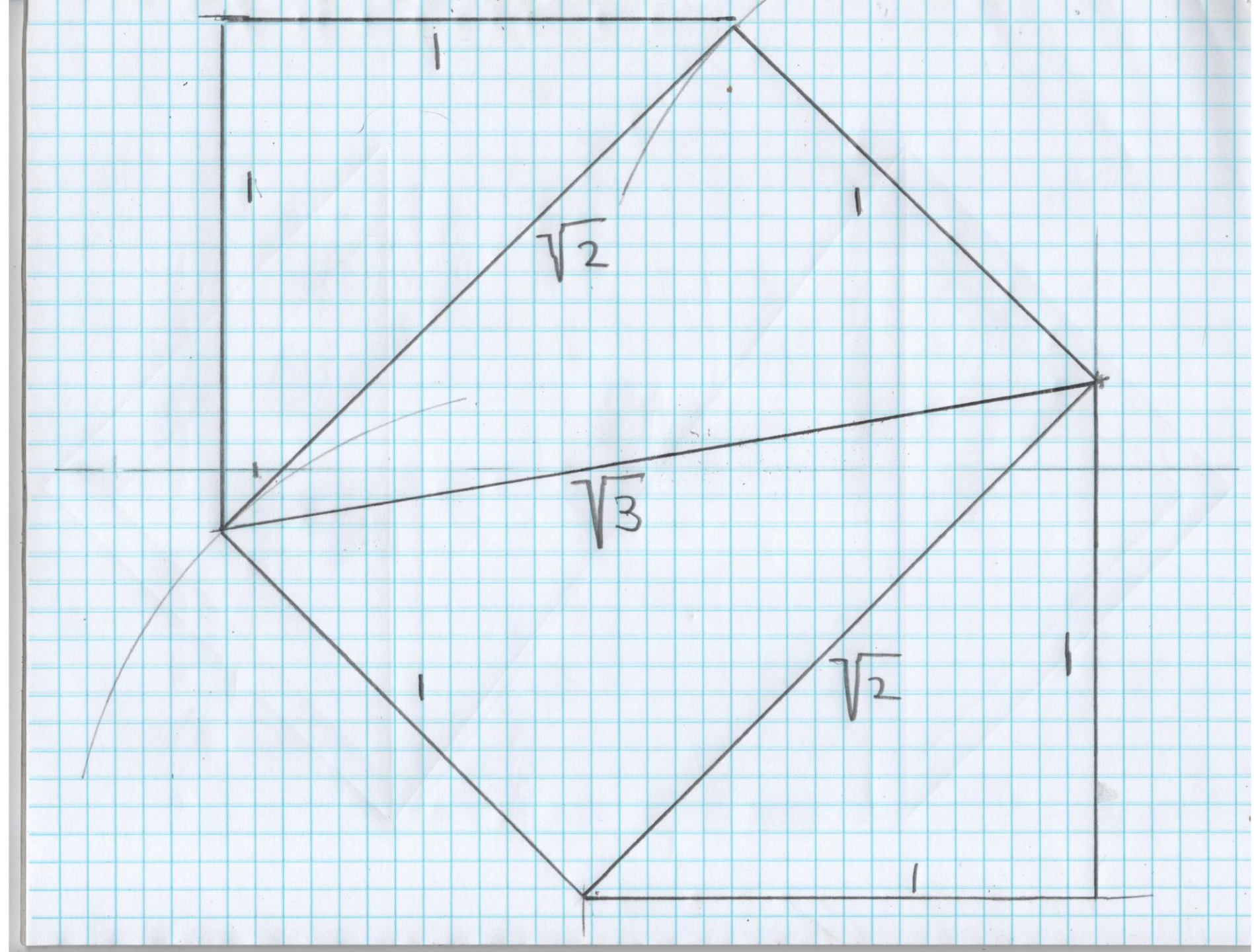
Visualizing and Encoding S3

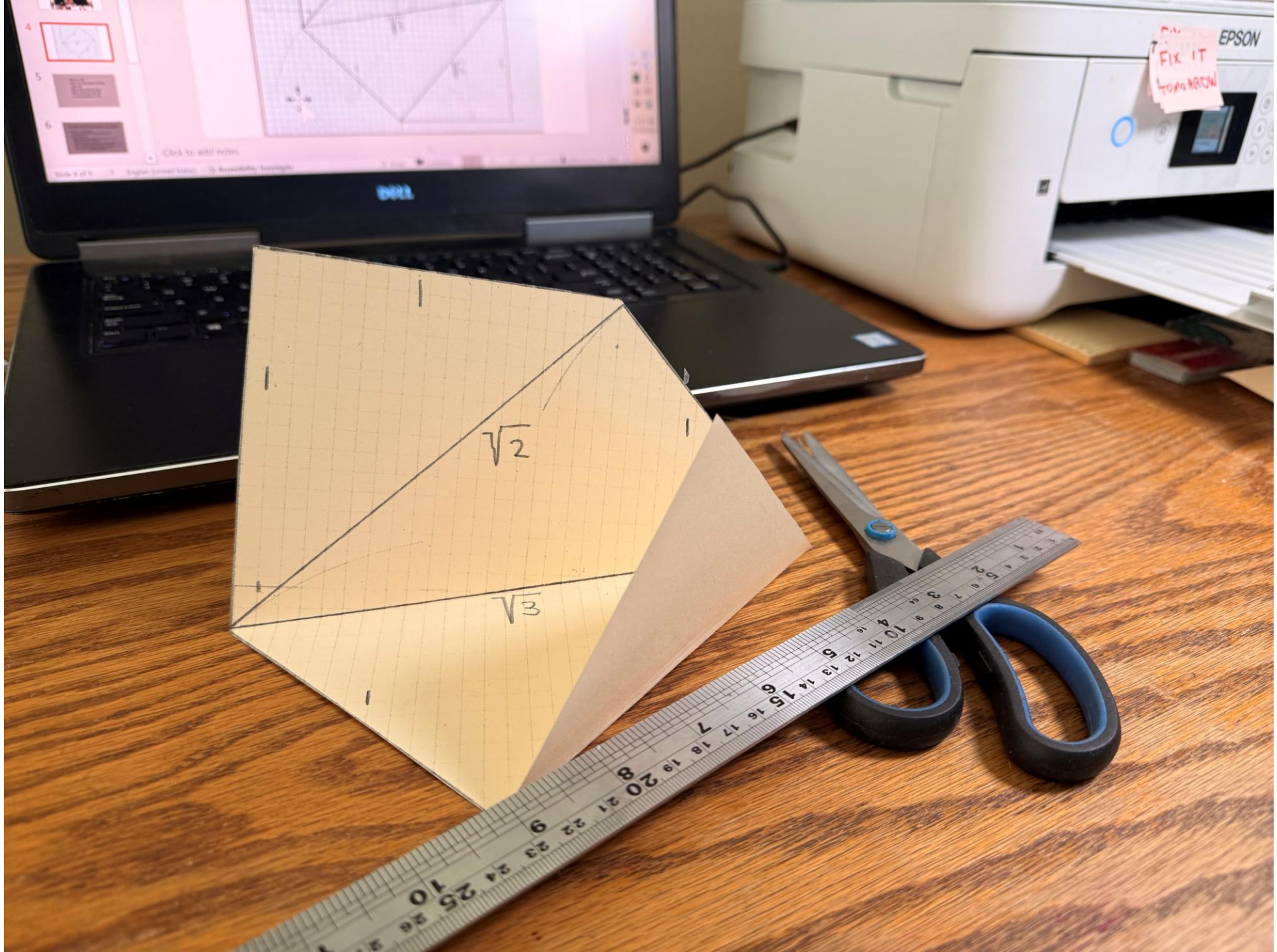
Richard Schwartz
Carson City, Nevada
richard1941@gmail.com



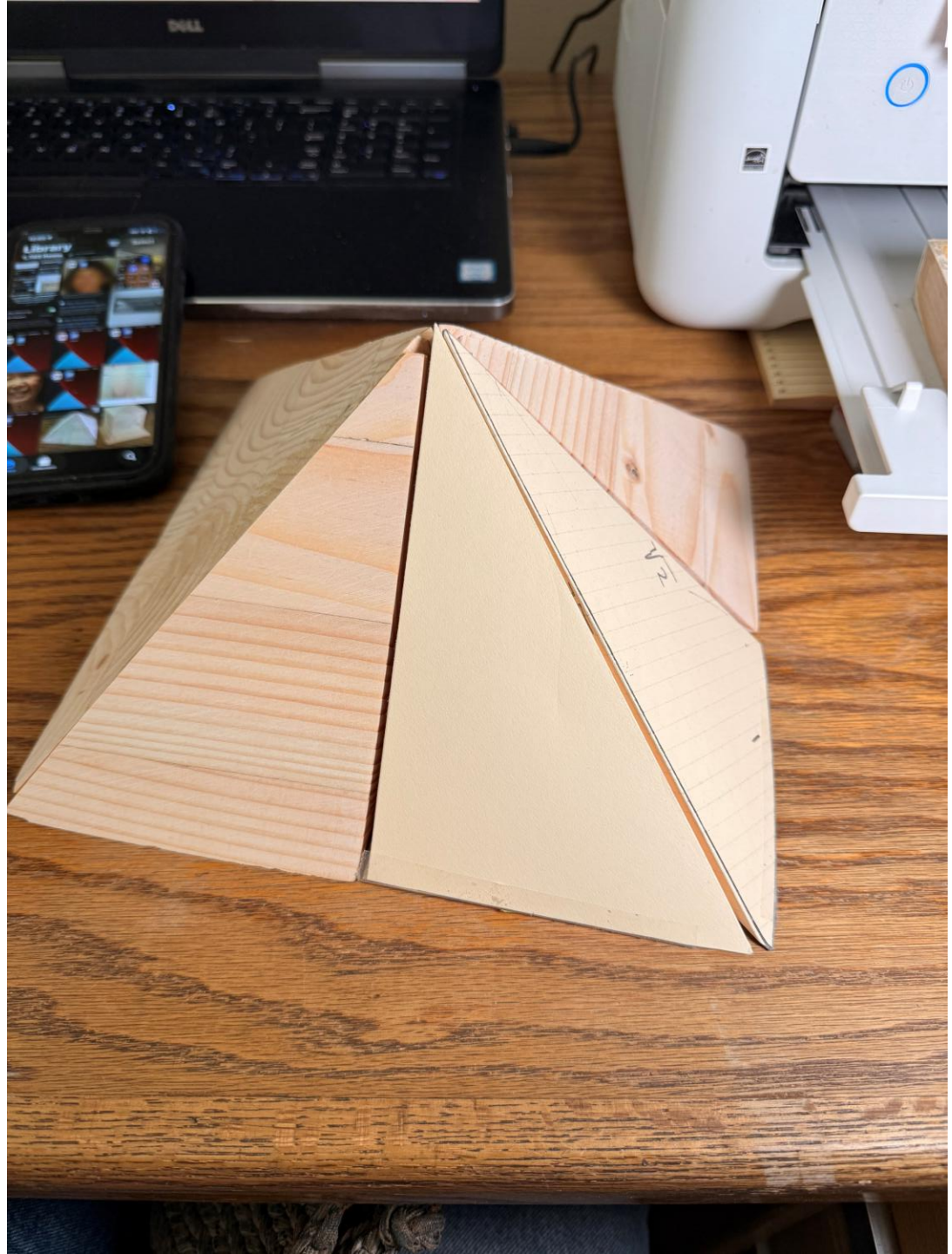





1
1
1
1
√2
√3
1
√2
1
1


1
1
1
√2
√3
1
EPSON
FIX IT
TOMORROW




DELL
Library



How was I made?

This is a major effort in woodshop technology beyond the scope of this presentation. A table saw (which I do not have) wood be helpful. If you must, the method of the previous slides is easier than cutting wood. (But don't step on them.)



What's wrong with me?

For some corners wood grain runs across a sharply pointed vertex.

Requires careful handling to avoid breaking point off.

- What is S3?
- How can elements of S3 be indexed?
- What does S3 look like?
- What about S4 and S5?
- A nasty problem in S8

S3 is the set of Permutations of 3 objects.

- S3 has 3! = 6 Elements
  - The elements are: 123, 132, 213, 231, 312, 321
- Indexing this set:
  - Notice how many numbers following the first are less; multiply that count by 2
  - Notice how many numbers following the second are less; multiply that count (0 or 1) by 1 and add.
  - The sum is the Lehmer Code, and integer from 0 to 5.
  - Note that this is **lexicographic** order and also is numerical order of the element names are taken as hex integers.

Operation Table for Permutations

		Y Lehmer Number					
Permutation	X	0	1	2	3	4	5
123	0	0	1	2	3	4	5
132	1	1	0	3	2	5	4
213	2	2	4	0	5	1	3
231	3	3	5	1	4	0	2
312	4	4	2	5	0	3	1
321	5	5	3	4	1	2	0

Mommy, Mommy !!!

“Guess what we learned in school today in new math!

3+1 =5.

I know it is true because Teacher proved it! Nobody else knows that; aren’t you glad we live in the CSSR, where the Supreme Leaders tell the Truth and nose rings are considered beautiful?

A Computer Science Problem:

- The way we compute the Lehmer code for a permutation is s similar to bubble sort.
- Is there a faster way to do it as with sorting algoritms like quicksort?
- Buble sort is known to be most efficient for lists up to length 7.
- 7! = 5040; we are not likely to be working with a histogram with that many bins. But what about other kinds of problems?

An Algebraic Aside

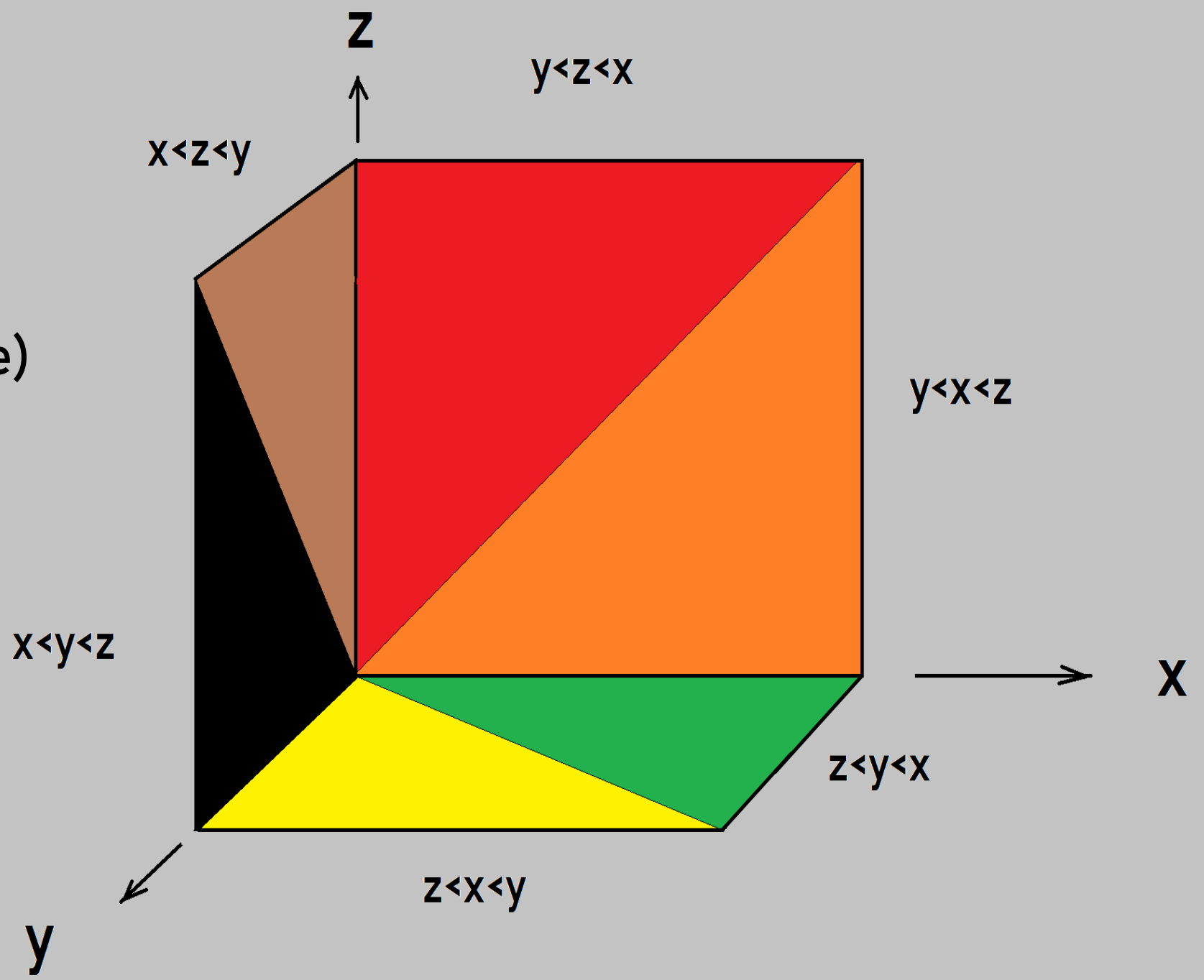
- The permutations under composition form a group of order 6.
- Five subgroups:
  - The Identity alone
  - {0,1}, {0,2}, {0,5}
  - {0,3,4}

Another Algebraic Aside

Every finite group is isomorphic to a subset of a permutation group.

So whatever we know about permutations is applicable to EVERYTHING!

(resistor color code)



Slice the cube on three planes

- x=y
- x=z
- y=z

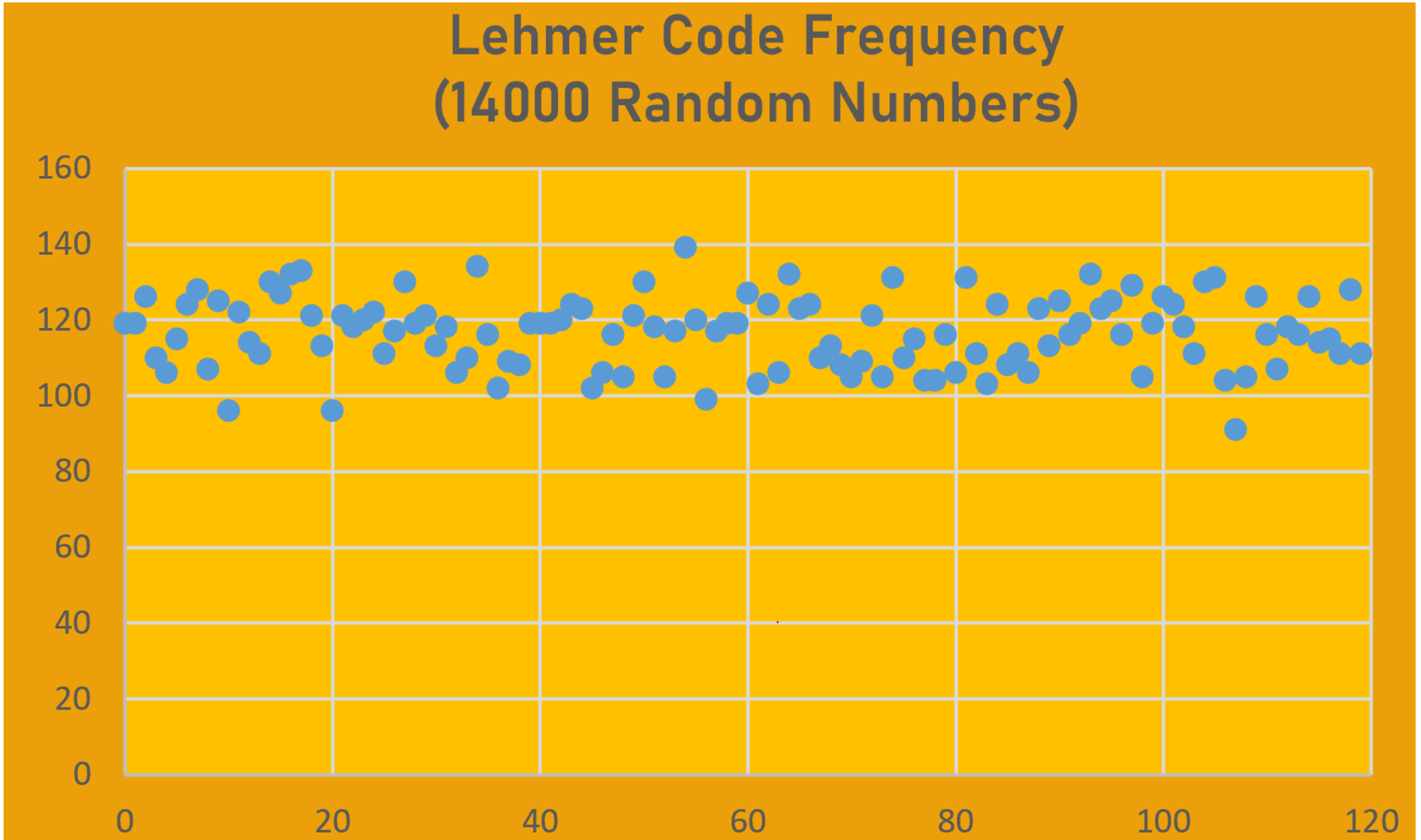
(Of course, in 4D hyperspace, there would be six hyperplanes. And in 5D there would be ten.)

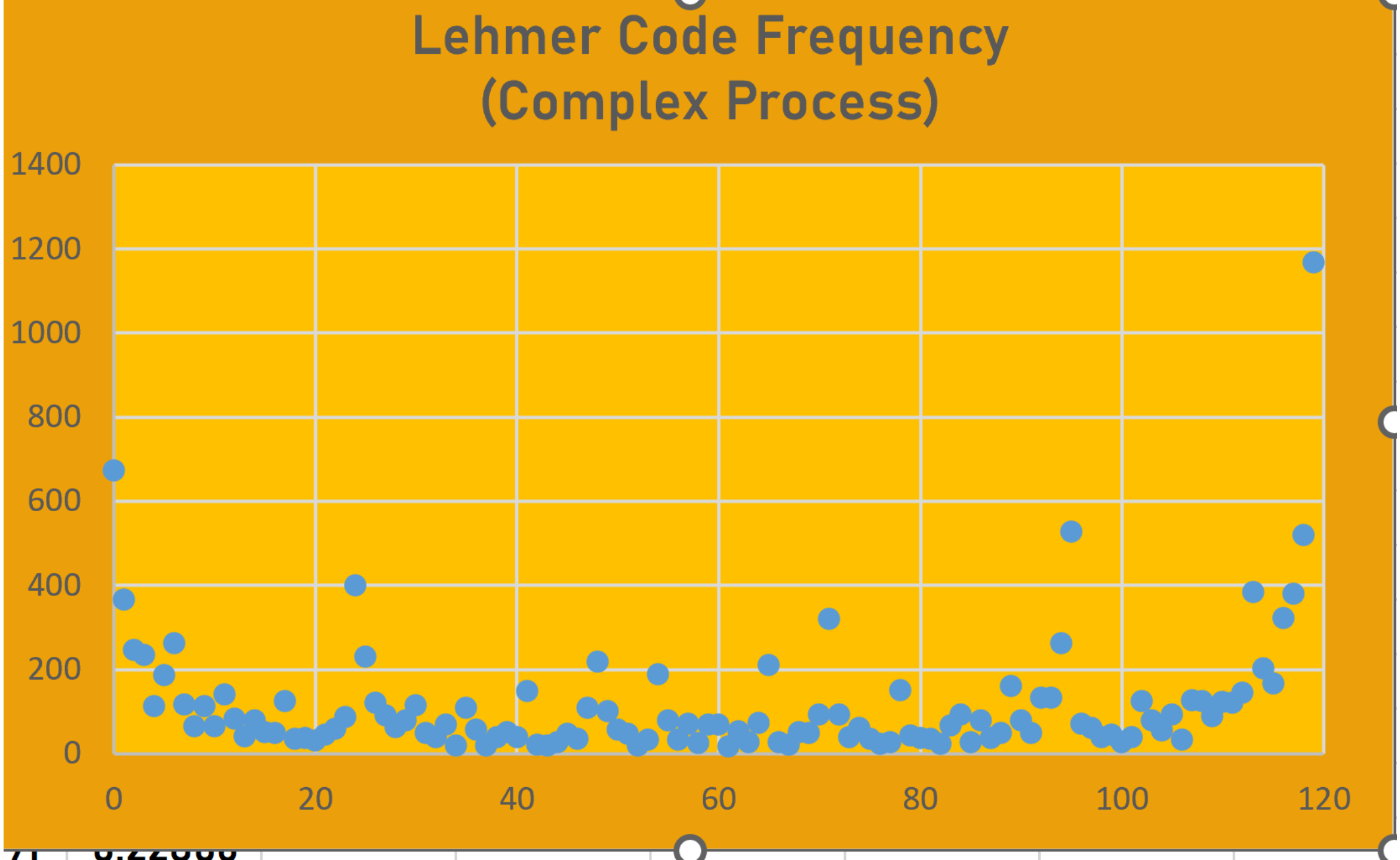
Some Observations

- Three of the pieces are the mirror image of the other three. These correspond to even and odd permutations.
- Three pieces may be selected to form a half cube; the remaining pieces also form a half cube.
- One of those three must be the mirror image of the other two.

Some Questions for the Curious . . .

- What is the maximum possible distance between two points in a hypercube?
- What is the distribution function of that distance, assuming the RNG is uniform?
- What is the maximum possible distance between two points in a single permutation in the hypercube?
- What is the distribution function of that distance, assuming the RNG is uniform?
- Can this be a new kind of RNG test?


Lehmer Code Frequency
(14000 Random Numbers)
160
140
120
100
80
60
40
20
0
0
20
40
60
80
100
120


Lehmer Code Frequency
(Complex Process)
1400
1200
1000
800
600
400
200
0
0
20
40
60
80
100
120

Any predictive value here?

Conclusion:

Be curious! Go out there and find a way to make those trusted RNG's FAIL . . .

...no matter how long it takes.

Special thanks to Paul Sellers, who inspired part of this project and provided invaluable guidance on how to do it.

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